Lecture 2

Accuracy of Machine Tools

Outline

• Definition of Accuracy
• Accuracy Measurement Standards
• Classification of Errors
• Definition of Errors
Definition of Accuracy

• **Accuracy** is the ability to tell the truth or:
  
  ➢ The maximum translational or rotational error between any two points in the machine's working volume.
  
  ➢ The deviation of the measured value from the ‘true’ value.
  
  ➢ Linear, planar, and volumetric accuracy can all be defined for a machine.

• **Repeatability** (Precision) is the ability to tell the same story over and over again or:
  
  ➢ The error between a number of successive attempts to move the machine to the same position.
  
  ➢ Repeatability is often considered to be the most important parameter of a computer controlled machine (or sensor).
  
  ➢ Often the intent is to map the errors and then compensate for them.

• Minimize static friction and thermal variants to get better repeatability.

• Note: mechanical accuracy for devices is costly, whereas repeatability is not expensive.
Definition of Accuracy

• **Resolution** is how detailed your story is
  - The smallest discernible change in the parameter of interest that can be detected by the instrument,
  - The smallest positional increment that can be commanded of a motion control system,
  - The smallest programmable step,
  - The smallest mechanical step the machine can make during point to point motion.

• Resolution gives a lower bound on the repeatability.

• Minimize static friction to get better resolution.
Accuracy vs. Precision

- good accuracy
  - poor precision
- poor accuracy
  - good precision
- good accuracy
  - good precision

Repeatability

- 66.27%
- 96.45%
- 98.73%
Accuracy, Precision and Standard Deviation

• A measurement can be precise, but may not be accurate.

• The standard deviation ($\sigma$) is a statistical measure of the precision in a series of repetitive $x_i$ measurements with $n$, the number of data, $x_i$ is each individual measurement, and the mean of all measurements.

• The value $x_i - \bar{x}$ is called the residual for each measurement.

\[
\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}}; \quad \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i
\]

Accuracy, Repeatability, Resolution

Curve Fitting:
One adjusting point: Linear
Two adjusting points: Bilinear
Three Points & above: two & higher order curve
- $e_j$ is the uncorrected systematic error, at the point $j$, with respect to the starting point of the calibration.
- $H_j$ is the uncorrected hysteresis error at point $j$. (backlash)
- $P_j$ is the unidirectional repeatability at the point $j$.
- $R_j$ is the bidirectional repeatability at the point $j$, which includes hysteresis effects, thus $R_j = P_j + H_j$. 2-11

**National Standards Organizations**

- American National Standards Institute (ANSI)
- British Standards Institute (BSI)
- Deutsches Institut für Normung (DIN)
- Japanese Industrial Standards (JIS)
- American Society of Mechanical Engineers (ASME)
- NMTBA (United States), ISO 230-1~9 (Europe), BSI BS 4656 Part 16 (British), VDI/DGQ 3441 (German), JIS B 6336-1986 (Japanese), and ASME B5.54-92 (USA).
Accuracy Measurement Standards

• ASME B5.54 *Methods for Performance Evaluation of Computer Numerically Controlled Machining Centers* which "establishes requirements and methods for specifying and testing the performance of [computer numerically controlled] CNC machining centers."

• ANSI/ASME B5.54 defines common terms, machine types, machining ranges (workzone), position resolution, and operating modes. It also addresses machine environmental requirements and responses. This standard provides tests for evaluating machine accuracy performance as a machine tool, the machine as a measuring machine with probes in the spindle, machine cutting performance and, optionally, the machining of test parts for the assessment of point-to-point machining capability and contouring capability.

Accuracy Measurement Standards

• ISO 230-1:1996
  ➢ *Test code for machine tools - Part 1: Geometric accuracy of machines operating under no-load or finishing conditions.*

• ISO 230-2:2006
  ➢ *Test code for machine tools - Part 2: Determination of accuracy and repeatability of positioning of numerically controlled axes.*

• ISO 230-3:2007
  ➢ *Test code for machine tools - Part 3: Determination of thermal effects.*

• ISO 230-4:2005
  ➢ *Test code for machine tools - Part 4: Circular tests for numerically controlled machine tools.*

• ISO 230-5:2000
  ➢ *Test code for machine tools - Part 5: Determination of the noise emission.*

• ISO 230-6:2002
  ➢ *Test code for machine tools - Part 6: Determination of positioning accuracy on body and face diagonals (Diagonal displacement tests).*

• ISO 230-7:2006
  ➢ *Test code for machine tools - Part 7: Geometric accuracy of axes of rotation.*

• ISO/TR 230-9:2005
  ➢ *Test code for machine tools – Part 9: Estimation of measurement uncertainty for machine tool tests according to series ISO 230, basic equations.*
Gage Accuracy Requirement for Errors Measuring in Machine Tools

• Gage accuracy requirement in factory
  ➢ gage accuracy requires a 4-to-1 ratio, machine-measuring accuracy must be four times more accurate than the specified part accuracy,

• Gage accuracy requirement in national standard laboratory
  ➢ according to a National Institute of Standards and Technology standard that gage accuracy requires a 10-to-1 ratio.

Equations of Error in ISO 230-2

• $E \uparrow$ or $E \downarrow$: Unidirectional Systematic Deviation of Positioning of an Axis ($E \uparrow$ or $E \downarrow$; 單軸單向系統偏差)

  Forward: $E \uparrow = \max[\bar{X}_i \uparrow] - \min[\bar{X}_i \uparrow]$  
  Backward: $E \downarrow = \max[\bar{X}_i \downarrow] - \min[\bar{X}_i \downarrow]$

  where $\bar{X}_i \uparrow = \frac{\sum_{ij} X_{ij} \uparrow}{n}$; $\bar{X}_i \downarrow = \frac{\sum_{ij} X_{ij} \downarrow}{n}$

  $n$ is the number of measured points
Equations of Error in ISO 230-2

- **E: Bidirectional Systematic Deviation of Positioning of an Axis** (單軸雙向系統偏差)
  \[ E = \max[\overline{X}_i \uparrow; \overline{X}_i \downarrow] - \min[\overline{X}_i \uparrow; \overline{X}_i \downarrow] \]

- **\( \sigma_i \): Standard Deviation** (標準差)
  \[ \sigma_i = \sqrt{\frac{1}{n-1} \sum (X_i - \overline{X}_i)^2} \]

  ➢ **Forward Direction**
  \[ \sigma_i \uparrow = \sqrt{\frac{1}{n-1} \sum (X_i \uparrow - \overline{X}_i \uparrow)^2} \]

  ➢ **Backward Direction**
  \[ \sigma_i \downarrow = \sqrt{\frac{1}{n-1} \sum (X_i \downarrow - \overline{X}_i \downarrow)^2} \]
Equations of Error in ISO 230-2

• R↑ or R↓: Unidirectional Repeatability of Positioning of an Axis (單軸單向最大重現性誤差值)

  ➢ Forward Direction
  
  \[ R \uparrow = \max[4\sigma_i \uparrow] \]

  ➢ Backward Direction
  
  \[ R \downarrow = \max[4\sigma_i \downarrow] \]

For High Precision

  ➢ 4 σ is replaced by 6 σ

Equations of Error in ISO 230-2

• R: Bidirectional Repeatability of Positioning of an Axis (單軸雙向重現性誤差值)

  \[ R = \max[2\sigma_i \uparrow + 2\sigma_i \downarrow + |B_i|; \ R \uparrow; \ R \downarrow] \]

where \( B_i = \bar{X}_i \uparrow - \bar{X}_i \downarrow \)

• For High Precision

  ➢ 2 σ is replaced by 3 σ
Equations of Error in ISO 230-2

• \( A \uparrow \text{ or } A \downarrow \): Unidirectional Accuracy of Positioning of an Axis (單軸單向最大精度誤差值)

- **Forward Direction**

\[
A \uparrow = \max[\bar{X}_i \uparrow + 2\sigma_i \uparrow] - \min[\bar{X}_i \uparrow - 2\sigma_i \uparrow]
\]

- **Backward Direction**

\[
A \downarrow = \max[\bar{X}_i \downarrow + 2\sigma_i \downarrow] - \min[\bar{X}_i \downarrow - 2\sigma_i \downarrow]
\]

For High Precision

► 4 \( \sigma \) is replaced by 6 \( \sigma \)

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Equations of Error in ISO 230-2

• Accuracy (A): Bidirectional Accuracy of Positioning of an Axis (準確度，單軸雙向精度誤差值):

\[
A = \max[\bar{X}_i \uparrow + 2\sigma_i \uparrow; \bar{X}_i \downarrow + 2\sigma_i \downarrow] - \min[\bar{X}_i \uparrow - 2\sigma_i \uparrow; \bar{X}_i \downarrow - 2\sigma_i \downarrow]
\]

For High Precision

► 4 \( \sigma \) is replaced by 6 \( \sigma \)
Errors in Machine Tools (1/4)

- Positioning error of each axis
- Straightness of each axis in its perpendicular axes
- Pitch, Yaw and Roll errors of each of the axes
- Squareness error between the axes
- Backlash error of each axis (Except for 21 Errors)
- Contouring error of each axis (Except for 21 Errors)

Errors in Machine Tools (2/4)

- The rigid body 21 errors for three axis machine tools include three each of the following errors:
  - linear displacement,
  - vertical straightness,
  - horizontal straightness,
  - roll angular,
  - pitch angular,
  - yaw angular,
  - and squareness.
Using a conventional laser interferometer for measuring the straightness and squareness errors requires a prohibitive amount of time, leading to the development of the body diagonal displacement method for a quick check as defined in the ASME B5.54 or ISO 230-6 standards.
### Classification of Errors

- **Accuracy and Repeatability are limited by:**
  - Geometric errors of all components
  - Kinematic errors
  - Load induced errors
  - Thermal errors
  - Dynamic errors
  - Calibration errors
  - Computational errors

- **Resolution is limited by**
  - Quality of sensors
  - Quality of control system
  - Friction (stick and slip effect)
  - Backlash
Geometric Errors (1/4)

- Geometric errors are those errors that exist in a machine tool from its basic design, the inaccuracies built-in during assembly and as a result of the components used on the machine. Because of that, they form one of the greatest sources of inaccuracy.
- These errors originate from the so-called “quasi-static” accuracy of surfaces moving relative to each other.

Geometric Errors (2/4)

- Quasi-static accuracy of surfaces moving relative to each other (e.g., linear or rotary motion axes):
  - Linear motion axis:
    - Pitch
    - Roll
    - Yaw
    - Straightness (2 components)
    - Linear displacement
Geometric Errors (3/4)

Rotary motion axis:

- Radial error motion (2 components in fixed coordinate frame, 1 component in rotating frame)
- Axial error motion
- Tilt motion (2 components)
- Angular motion about axis of rotation

Radial error motion

- Radial error motion - Positioning error of the rotary stage in the horizontal direction when the tabletop is oriented in the horizontal plane. Radial runout is defined as the total indicated (TIR) reading on a spherical ball positioned 50 mm above the tabletop and centered on the axis of rotation.
Radial error motion


Axial error motion

• *Axial error motion* - Error of the rotary stage axis of rotation in the vertical direction when the stage is oriented in the horizontal plane. Axial runout is defined as the total indicated reading (TIR) on a spherical ball positioned 50 mm above the tabletop and centered on the axis of rotation.
Axial error motion


Tilt error motion

- *Tilt error motion* - Wobble is defined as the angular error between the actual axis of rotation and the theoretical axis of rotation.
Geometric Errors (4/4)

- **Geometric errors** can be smooth and continuous or they could show hysteresis (遲滞后) or random behavior.
- These errors are affected by **factors** like surface straightness, surface roughness, bearing pre-loads etc.
Hysteresis Effect (遲滯效應)

Hysteresis error

- *Hysteresis error* - A deviation between the actual and commanded position at the point of interest caused by elastic forces in the motion system. Hysteresis also affects bi-directional repeatability. Accuracy and repeatability errors caused by hysteresis for Aerotech rotary stages are accounted for in the stage specification tables. Elastic forces in the machine base, load, and load coupling hardware must also be examined and minimized for optimal performance.
Backlash error

- *Backlash error* - An error in positioning caused by the reversal of travel direction. Backlash is the portion of commanded motion that produces no change in position upon reversal of travel direction. Backlash is caused by clearance between elements in the drive train. As the clearance increases, the amount of input required to produce motion is greater. This increase in clearance results in increased backlash error. Backlash also affects repeatability. Unidirectional repeatability refers to the repeatability when approached from the same direction. It does not take into account the effects of backlash. Bidirectional repeatability specifies the repeatability when approached from any direction and includes the effects of backlash. ADR and AOM360 series tables are direct-drive devices and therefore have zero backlash.

Feedback inaccuracy

- *Feedback inaccuracy* - Imperfections in the operation of the encoder such as non-uniform division of the grating scale, imperfections in the photodetector signal, interpolator errors, hysteresis, friction, and noise can affect the positioning capabilities of the rotary stage. For a rotary equipment, the accuracy and repeatability information in the specification tables takes all of these errors into account.
Errors in form

Other Errors

• Kinematic Errors
  
  ➢ *Errors in an axis's trajectory that are caused by misaligned or improperly sized components*
  
  ✈ Squareness between axes
  ✈ Parallelism between axes
  ✈ Error motions in a closed kinematic chain
  ✈ External load induced errors:

  ➢ *Errors due to deformation of components*
  
  ✈ Gravity load induced errors
  ✈ Cutting/probing force induced errors
  ✈ Axis acceleration load induced errors
Kinematic errors (1/3)

• Kinematic errors are due to the relative motion errors of several moving machine components that need to move with precise functional requirements.
• These errors are particularly significant during the combined motion of different axes.
• Such errors occur during linear, circular or other types of interpolation algorithms and are more obvious during actual machining.

Kinematic errors (2/3)

• Errors in motion due to alignment.
Kinematic errors (3/3)

Errors in motion due to shape:

• Improper offsets (translational) between components.
  ➢ Spindle axis set too high above tailstock axis on a lathe.

• Improper component dimension.
  ➢ Linkage length.
  ➢ Bearing location on a kinematic vee and flat system.

Thermal Errors (1/4)

• Thermal errors due to thermal factors account for 40-70% of the total dimensional and shape errors of a workpiece in precision engineering.

• Six sources of thermal influence are identified:
  (i) heat from the cutting process,
  (ii) heat generated by the machine,
  (iii) heating or cooling provided by the cooling systems,
  (iv) heating or cooling influence of the room,
  (v) the effect of people, and
  (vi) thermal memory from any previous environment
Thermal Errors (2/4)

• This heat causes relative expansion of the various elements of the machine tool leading to inaccurate positioning of the cutting tool. Consequently errors due to spindle growth, thermal expansion of the ballscrews and thermal distortion of the bed are generated at the tool tip.

• As heat generation at contact points is unavoidable, this source of error is one of the most difficult to eliminate completely. In the manufacture of precision components, error due to thermal deformation of the machine elements plays a vital role in limiting the accuracy of the part produced.

Thermal Errors (3/4)

• Mean temperature other than 68 °F (20 °C).
  ➢ Gradients in environment's temperature
  ➢ Errors caused by thermal expansion of elements:
    ☝️ External heat sources:
      – Mean temperature of the room
      – Sun shining through the window onto the machine
      – Nearby machine's hot air vent
      – Overhead lights
      – Operator's body heat
    ☝️ Internal heat sources:
      – Motors
      – Bearings
      – Machining process
      – Pumps
      – Expansion of compressed fluids
      – Coolant
Thermal Errors (4/4)

- Design strategies:
  - Isolate heat sources and temperature control the system.
  - Maximize conductivity, or insulate
  - Combine one of above with mapping and real time error correction.
  - May be difficult for thermal errors because of changing boundary conditions.

Thermal Sources (1/6)

Description

- Speed creates heat, which impacts accuracy and repeatability.
- This simple equation becomes more problematic with the longer cycle times and higher speeds and feeds of high-speed applications.
- However, most applications focus so much on keeping heat away from the workpiece that other areas impacted by thermal distortion are overlooked.
- Thermal stability is maintained by improved heat dissipation throughout the machining center's various components.
Thermal Sources (2/6)

Sources

1. Spindle:

- High-speed spindles can experience growth due to heat from friction running at high rpms and require a long saturation period before they stabilize.
- Too much heat will compromise accuracy and can cause failure.
- Makino's patented core cooling and under-race lubrication system cools the spindle from the inside out to minimize heat and growth for a shorter saturation period.
- The cooling system circulates Makino's spindle oil through the center of the rotating spindle. At high rpms, centrifugal force draws the lubricant outward through the spindle circulating through holes in the inner bearing races to lubricate and chill the bearings.

Thermal Sources (3/6)

Makino's patented core cooling and under-race lubrication system in high speed spindle.
2. **Ballscrews:** ballscrew heat from high feedrate levels by forcing chilled oil through the core.

3. **Hot chips,** and chip-heated coolant are also kept away by shields installed to protect the machining center.

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4. **Support Components:**
   - *Running normal operations,* miscellaneous machining center components can also become heat sources.
   - *The location and design of pumps, motors, hydraulics and magnetics* are key.
   - *Some machining centers are designed so these components are mounted at the rear of the machine with a dead air space isolating them. To ensure this heat cannot impact the machining center,* a radiator cooling system is sometimes used to wrap the machining center column.
5. Environment:

- But even with these safeguards in place, the shop environment must be checked for external heat sources.
- From sunlight on the machining center to external heat on the shop floor—any increase or decrease in temperature can negatively impact accuracy and repeatability.

Dynamic Errors

- Errors caused by *vibration* or *control processes*:
  - **Vibration:**
    - External environment (usually through the ground)
    - Cutting process
    - Rotating masses
  - **Control system:**
    - Algorithm type (e.g., PID, adaptive, etc.)
    - Stick-slip friction
    - Varying mass
    - Varying stiffness
  - **Switching amplifiers**
    - Servo loop frequency excites a natural mode of the machine
Calibration Errors

- Errors associated with sensors:
  - *Intrinsic accuracy*
  - *Interpolation*
  - *Mounting errors:*
    - Position
    - Mounting stress
  - *Calibration (error associated with the mastering process)*

Additional Errors

- Computational errors:
  - Error introduced in the analysis algorithms
  - Rounding off errors due to hardware
- Additional sources of error (often very difficult to model):
  - Humidity
  - Loose Joints
  - Dirt
- Variations in supply systems:
  - Electricity
  - Fluid pressure
  - Operator inattention
  - Fluid supply cleanliness
- Operators
Cutting-force induced errors (1/2)

- The **dynamic stiffness** of all the components of the machine tool (namely the bed, column, etc.) that are within the cutting loop of the machine is responsible for errors caused as a result of the cutting action.
- This is one of the major sources of error in metal-cutting machines as the force involved in the cutting action is considerable.
- As a result of these forces, the position of the tool tip with respect to the workpiece varies on account of the distortion of the various elements of the machine.
Cutting-force induced errors (2/2)

- Depending on the stiffness of the structure under the particular cutting conditions, the accuracy of the machine tool could vary.
- Therefore, for a machine with a given stiffness, a heavy cut would generally produce more inaccurate components than a light cut.

Error Assessment and Budgeting

- Given all the different types of errors that can affect all different components:
  - Keeping track of all the errors is such a daunting task:
    - Most engineers don't bother and use "experience" to guide the design.
    - It is left up to manufacturing and service to work the bugs out.
  - This seems to be a major source of reliability and performance problems.
- The solution to a successful project is a good budget:
  - A project requires a good financial budget to make it feasible.
  - A project requires a good time budget to make it feasible.
  - A project requires a good error budget to make it feasible.
- In order to make a good error budget for the system, a good mathematical model is needed.
Glossary

• **Home reference mark** - the location on a linear scale which provides an independent electrical output to locate the home or zero reference position.

• **Step** - in a stepping motor drive, the minimum rotational movement allowed by the system.

• **Velocity** - the rate of change of position with time.

• **Holding torque** - the amount of torque available from a stepping motor when the windings are energized but the rotor is stopped.

• **Lead** - the distance traveled by the leadscrew nut for each revolution of the leadscrew.

• **Leadscrew pitch** - the number of revolutions required to advance the leadscrew nut one inch.

Glossary

• **Squareness (Orthogonality)** - the error from true 90-degree perpendicularity of two axes.

• **Open loop positioning** - a positioning system which does not employ feedback information.

• **Closed-loop positioning** - a positioning system that employs an external feedback element to measure stage position. Typically, a linear encoder mounted to the axis will eliminate hysteresis, backlash, and leadscrew errors.
Glossary

• **Linear positioning accuracy** - the error between the desired move and the actual position achieved by a linear positioning component or stage system.

• **Error** - the difference between the actual and the desired condition.

• **Accuracy** - the deviation from the exact value of the desired position or velocity.

• **Repeatability**
  - a. **Uni-directional repeatability** - the ability of a system to repeat to a desired location approaching that location from the same direction each time.
  - b. **Bi-directional repeatability** - the ability of a system to repeat to a desired location approaching that location from both plus and minus directions.

• **Resolution** - the smallest incremental positioning move that a system can achieve or display.

Glossary

• **Backlash**
  - The amount of free play or clearance between two interactive components in a drive train or leadscrew, often referred to as a “dead-band” when the motion direction is reversed.
  - Backlash error is an error in positioning caused by the reversal of travel direction. Backlash is the portion of commanded motion that produces no change in position upon reversal of travel direction. Backlash is caused by clearance between elements in the drive train.

• **Flatness of travel**
  - Flatness is a deviation from the true line of travel perpendicular to the direction of travel in the vertical plane.
  - Flatness is the vertical deviation of a single point moving horizontally along a straight line.
  - Flatness errors are caused by a combination of roll and pitch errors.

• **Straightness of travel**
  - Straightness is a deviation from the true line of travel perpendicular to the direction of travel in the horizontal plane.
  - Straightness is the horizontal deviation of a single point moving horizontally along a straight line.
  - Straightness errors are caused by yaw and roll errors.
**Glossary**

- Rotation around the front-to-back axis is called roll.
- Rotation around the side-to-side axis is called pitch.
- Rotation around the vertical axis is called yaw.
Yaw, Pitch and Roll

- The 3D rotations are made in following order:
  - Roll (Z axe rotation)
  - Yaw (Y axe rotation)
  - Pitch (X axe rotation)

http://www.eecs.berkeley.edu/~jimy/research/trk300/angles.html
Glossary

• Pitch - rotation about the horizontal axis perpendicular to the axis of travel.

  ➢ Pitch is a rotation around an axis in the horizontal plane perpendicular to the direction of travel. If the position of interest being measured is not located at the center of rotation, then the pitch rotation will cause an Abbe error in two dimensions.

  ➢ For the X-axis, a pitch rotation will cause an Abbe error in both the X and Z direction. For the Y-axis, a pitch rotation will cause an Abbe error in both the Y and Z direction.

  ➢ The magnitude of these errors can be determined by multiplying the length of the offset distance by the sine and 1-cosine of the rotational angle.

\[ E_x = A_0 \sin \theta \quad E_z = A_0 (1 - \cos \theta) \quad A_0 : \text{offset distance} \]

Glossary

• Roll - rotation about the axis of movement while translating along that axis.

  ➢ Roll is a rotation around an axis in the horizontal plane parallel to the direction of travel. If the position of interest being measured is not located at the center of rotation, then the roll rotation will cause an Abbe error in two dimensions.

  ➢ For the X-axis, a roll rotation will cause an Abbe error in both the Y and Z direction. For the Y-axis, a roll rotation will cause an Abbe error in both the X and Z direction.

  ➢ The magnitude of these errors can be calculated by multiplying the length of the offset distance by the sine and cosine of the rotational angle.
Glossary

• Yaw - rotation about the vertical axis which is perpendicular to the axis of travel.
  ➢ *Yaw is a rotation around an axis in the vertical plane perpendicular to the direction of travel. If the position of interest being measured is not located at the center of rotation, then the yaw rotation will cause an Abbe error in two dimensions.*
  ➢ *For X- or Y-axis stages, yaw rotation will cause an Abbe error in both the X and Y direction.*
  ➢ *The magnitude of these positioning errors can be calculated by multiplying the length of the offset distance by the sine and cosine of the rotational angle.*

Calculate of Roll, Yaw, Pitch
(Given angles at three directions)

• Calculation:
  ➢ $R_x$ (Roll): Rotation about X by $x$ radians
  ➢ $R_y$ (Yaw): Rotation about Y by $y$ radians
  ➢ $R_z$ (Pitch): Rotation about Z by $z$ radians
Calculate of Roll, Yaw, Pitch
(Given angles at three directions)

\[
R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(x) & \sin(x) \\ 0 & -\sin(x) & \cos(x) \end{bmatrix}, \quad R_y = \begin{bmatrix} \cos(y) & 0 & -\sin(y) \\ 0 & 1 & 0 \\ \sin(y) & 0 & \cos(y) \end{bmatrix}, \quad R_z = \begin{bmatrix} \cos(z) & \sin(z) & 0 \\ -\sin(z) & \cos(z) & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

For Example:

\[
R_z R_x R_y = \begin{bmatrix} \cos(z) + \sin(z)\sin(x)\sin(y) & \sin(z)\cos(x) & \cos(z)\sin(y) + \sin(z)\sin(x)\sin(y) & 0 \\ -\sin(z)\cos(y) + \cos(z)\sin(x)\sin(y) & \cos(z)\cos(x) & \sin(z)\sin(y) + \cos(z)\sin(x)\cos(y) & 0 \\ \cos(x)\sin(y) & -\sin(x) & \cos(x)\cos(y) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]
Calculate of Roll, Yaw, Pitch

(Given errors at three directions)

\[
\begin{align*}
\text{Pitch} &= \sin^{-1}(xl_y) \\
\text{Roll} &= \sin^{-1}(\cos(\text{Pitch}) \cdot xl_x) \\
\text{Yaw} &= \tan^{-1}\left(\frac{mx \cdot \cos(\text{Pitch}) - my \cdot \sin(\text{Pitch}) \cdot \sin(\text{Roll}) - mz \cdot \sin(\text{Pitch}) \cdot \cos(\text{Roll})}{mx \cdot \cos(\text{Roll}) - mz \cdot \sin(\text{Roll})}\right)
\end{align*}
\]

where \(xl_x, xl_y\) are the values taken from the X-, and Y-axis accelerometers; \(mx, my, mz\) are the values of the X-, Y-, and Z-axis sensors.

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Glossary

- **Concentricity** - the difference between a rotating member is centerline and the actual true centerline of rotation.
- **Encoder error** (編碼器誤差)
  - Imperfections in the operation of the encoder such as absolute scale length, non-uniform division of the grating scale, imperfections in the photo-detector signal, interpolator errors, hysteresis, friction, and noise can affect the positioning capabilities of the linear translation stage. The accuracy and repeatability information in the specification tables takes all of these errors into account except absolute scale length. Absolute scale length is affected by thermal expansion of the encoder scale. Temperature considerations must be accounted for during system design and specification.
Glossary

• Hysteresis error (遲滯誤差)
  ➢ Hysteresis error is lost movement in the absolute position of an object when motion is reversed. It is caused by accumulated forces in elastic materials.
  ➢ Hysteresis error is a deviation between the actual and commanded position at the point of interest caused by elastic forces in the motion system.
  ➢ Hysteresis also affects bi-directional repeatability.

Abbe error (阿貝誤差) (1/4)

• Abbe error (阿貝誤差)
  ➢ Displacement error caused by angular errors in bearing ways and an offset distance between the point of interest and the drive mechanism (ball screw) or feedback mechanism (linear encoder).

指由於量具之量測軸與待測零件之尺寸軸的不一致，因量具的彈性變形所產生之量測誤差值稱之。
Abbe error (2/4)

\[
\text{Abbe Error} = \text{offset distance} \times \tan \theta = A_0 \cdot \tan \theta
\]

Abbe error (3/4)

High quality translation stages have sub-micron runout errors with angular deviations on the order of 100–150 µradians.
Cosine Error (1/2)

- **Cosine error** is most typically seen with test-style indicators and lever type electronic probes doing run-out and concentricity checks on shafts and bores; or, in engineering and tool making, doing checks of parallelism and alignment of flat faces.

- Cosine error results from *an angular misalignment* between the motion of a positioning table, and the accuracy determining element (leadscrew, encoder, or laser interferometer beam path).
Cosine Error (2/2)

\[ E = L_s - L = L_s (1 - \cos \theta) \]

Example

Abbé Offset = H
(Distance between Workpiece and Linear Encoder)

Abbé Error = H \Theta
(Difference in arc length between AB and MM')

Length of Arc AB = r \Theta
Length of Arc MM' = (r - H) \Theta

AB - MM' = r \Theta - (r - H) \Theta
AB - MM' = r \Theta - [r \Theta - H \Theta]
AB - MM' = r \Theta + r \Theta + H \Theta
AB - MM' = H \Theta
Errors from Motor (1/3)

- Positioning errors
  - They are caused by errors in the position detecting scale and servo system in the case of a closed-loop type NC (linear scale feedback type NC). For a semi-closed-loop type NC (encoder feedback type NC), they are caused by errors in the servo control system and the ball screw driving mechanism (nut, ball screw, coupling, servo motor).

Errors from Motor (2/3)

- Straightness errors, Squareness errors
  - They are caused by profile errors of the bed, column, saddle and their guide ways in the condition of the machine tool at installation.

- Angular motion errors I
  - Same as straightness/squareness errors, they are caused by profile errors of bed, column, saddle and their guide ways at the condition the machine tool was installed.
Errors from Motor (3/3)

• Angular motion error II
  - The magnitudes of angular motion errors are determined by the magnitudes of the moment added to the sliders by the gravity, counter balance force, ball screw driving force, and sliding friction during the motion of sliders. They are also determined by the rigidity of the guide ways that restrict the sliders. Inertial force should also be considered when the acceleration of feed motion is very high.

Origins of motion errors in NC machine tools

Positioning Errors
  • Errors in the scaling system
    - uniform expansion or contraction of the linear scale, cyclic error and local error.
    - Thermal expansion and distortion errors, where temperature change causes the leadscrew to grow or the temperature gradient to distort machine geometry, also affect the positioning errors.
  • Errors in the ball screw driving system
    - uniform expansion or contraction of the ball screw, pitch error, whirling, lost-motion, backlash, tilling of the thrust bearing: errors in coupling, transmission gear, timing belt, etc.
  • Errors in the servo control system
    - stick-motion, stick-slip, inadequate pitch error compensation, inadequate backlash compensation, reduction in radius during circular interpolation motion due to response lag, mismatching of position loop gain, noise in detectors.
Origins of motion errors in NC machine tools

• Straightness errors, squareness errors
  ➢ Straightness errors and squareness errors of guide ways.
  ➢ Straightness errors, caused when the guide way is not perfectly straight, usually because of weight shifting or overhanging during axis travel, may lead to positioning errors.

• Angular motion errors I
  ➢ Straightness errors and parallelity errors of guide ways.

• Angular motion errors II
  ➢ Asymmetrical guide way and ball screw, counter balance, shift of weight, levitation of slider, etc.

Classification of motion error origins

• Errors in positioning mechanism
  ➢ (a) (first order, second order) uniform expansion or contraction of the ball screw and linear scale
  ➢ (b) cyclic error of the ball screw, linear scale, etc.
  ➢ (c) noise in detectors
  ➢ (d) backlash
  ➢ (e) backlash compensation
  ➢ (f) pitch error compensation
Classification of motion error origins

• Profile errors of guide way
  ➢ (g) squareness errors between 2 axes
  ➢ (h) straightness errors
    ◦ (h-1) produced during manufacturing and assembling
    ◦ (h-2) due to improper installation of base
    ◦ (h-3) due to shift of weight
  ➢ (i) moment
    ◦ (i-l) rolling of vertical axis
    ◦ (i-2) pitching of vertical axis
    ◦ (i-3) yawing of vertical axis
    ◦ (i-4) yawing of vertical axis with pre-compensated geometry
    ◦ (i-5) yawing of horizontal axis
  ➢ (j) parallelity error
  ➢ (k) collision of hose, friction of the sliding cover

• Feedrate dependent errors
  ➢ (l) lost motion
  ➢ (m) stick motion
  ➢ (n) stick-slip
  ➢ (o) mismatching of position loop gain
  ➢ (p) decrease in radius of circular interpolation motion due to response lag in servo system
  ➢ (q) vibration of hydraulic valve, chain, etc.
  ➢ (r) levitation of sliders due to dynamic pressure
First Order Uniform Expansion or Contraction of Linear Scale

uniform expansion
or contraction

linear scale
slider

Second Order Uniform Expansion or Contraction of Linear Scale

Q

expansion
or contraction

linear scale
slider
Second Order Straightness Error of Guide Way

Elastic Deformation Caused by Improper Installation of Basement
Elastic Deformation Caused by Shift of Weight

Movement of Spindle Head in the Yawing

First Order function of the Y Coordinate

Full Line: Upward
Dotted Line: Downward
Movement of Spindle Head in the Yawing

Second Order function of the Y Coordinate

Full Line: Upward
Dotted Line: Downward

Parallellity Error

(a) guide ways without parallellity error
(b) guide ways with parallellity error
Yawing of Narrow Guide

Lost Motion
Lost Motion-Stick-Slip

Machine Design is Key to Volumetric Accuracy

- Based on their experience in the field, the leveling systems as the design characteristic usually impacting volumetric accuracy.
- High-performance machining centers utilize a three-point leveling system and incorporate much thicker castings to carry the weight and maintain the accuracy between the leveling points.