Lecture 16

Externally Pressurized Fluid Film Bearings
(Hydrostatic Bearing)

Outline

• General characteristics
• Hydrostatic bearings
• Aerostatic bearings
General Characteristics

• Lack of mechanical contact between elements causes error motions to be small, and harmonics quickly die out:

![Graph showing radial error motion versus frequency in Hz.]

• Hydrodynamic bearings operate based on the principle that:
  - Viscous fluids are dragged between bodies as they move past each other, so the rotating shaft acts like a pump.
  - The pressure gradient is limited, and so is the load capacity and stiffness.
  - They are the simplest of bearings.

• Hydrostatic and aerostatic bearings use an external pump to supply pressurized fluid to the bearing:
  - Metered flow to each side of the bearing creates a pressure differential proportional to the displacement.
  - Load capacity and stiffness can be very high.
  - They require the expense of a clean pressure supply system.
General Characteristics

• Capabilities and applications for externally pressurized bearings:

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Mixed phase</th>
<th>Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capillary, orifice, slot, or diaphragm restricted</td>
<td>Liquid/steam when slot restricted</td>
<td>Porous, orifice, or slot restricted</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Highly suitable for machine tools</th>
<th>If correctly designed, bearing will operate in either media and where contamination is unacceptable</th>
<th>Highly suitable for textile machines</th>
</tr>
</thead>
</table>

- High load capacity
- Moderate load capacity: grinding spindles, diamond turning spindles, and instruments

<table>
<thead>
<tr>
<th>Very high stiffness</th>
<th>High stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very high damping</td>
<td>Moderate-low damping</td>
</tr>
<tr>
<td>Low friction at low speed</td>
<td>Very low friction at all speeds</td>
</tr>
</tbody>
</table>

For any of these bearings, a designer can rapidly check feasibility with the following formulae:

\[ F_{\text{load capacity}} = \frac{A_{\text{bearing area}} P_{\text{supply pressure}}}{2} \]

\[ K_{\text{stiffness}} = \frac{F_{\text{load capacity}}}{h_{\text{radial bearing gap}}} \]
Hydrostatic Bearings

• Applied loads
  ➢ Large surface area allows for high load capacity.
  ➢ Virtually insensitive to crashes.

• Accuracy
  ➢ Axial: limited only by the drive system.
  ➢ Lateral: limited by the rails and isolation from the pressure source.

• Preload
  ➢ Most designs are inherently preloaded.
Hydrostatic Bearings

• **Stiffness**
  - *Easily made many times greater than other components in the machine.*
  - *Dynamic stiffness is very high due to squeeze film damping.*

• **Vibration and shock resistance**
  - *Excellent for liquid bearings.*
  - *Modest-to-poor for gas bearings.*

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Hydrostatic Bearings

• **Damping capability**
  - *Excellent normal to direction of motion, due to squeeze film damping.*
  - *Low along direction of motion.*
  - *Bearing area, gap, and stiffness must be considered to maximize squeeze film damping.*
Hydrostatic Bearings

- Squeeze film damping greatly affects the dynamic stiffness.

Hydrostatic Bearings

- Friction
  - Zero static friction.
  - Dynamic friction depends on gap and fluid viscosity.

- Thermal performance
  - Finite dynamic friction coefficient generates heat.
  - Fluid flowing at pressure released to atmospheric pressure shears and generates heat equal to pump power.
    - A cooler is often needed to control fluid temperature.
  - Expanding gas creates cooling (Joule Thompson cooling).

- Environmental sensitivity
  - Very intolerant of dirt.
    - Where the fluid has to flow past a tiny gap (a capillary or an orifice), it can clog.
    - Gear pump flow-dividers and self-compensating bearings are "self cleaning."
  - A particle lodged in a small gap can score the bearing or the rail.
Hydrostatic Bearings

• **Support equipment**
  - *Pumps:*
  - Screw-type pumps are most quiet.
  - Piston pumps are noisiest
  - Accumulators and pressure relief valves are needed to keep pressure pulsations from increasing error motions.
  - Pumps generate heat!
  - If linear motors are used on the machine, they will require an order-of-magnitude more cooling than the hydrostatic bearings!

Hydrostatic Bearings

• **Filters:**
  - Air bearings ideally are fitted with desiccant dryers.
  - Fluid bearings require filters.
  - Water bearings require fluid chemistry control.
  - Centrifugal filters work well, but are expensive.
  - Cartridge filters must be changed.
Theory of operation for plane opposed bearings with fixed compensation

- Fluid flow into the bearing is regulated (R) by a resistance. \((P=QR)\) Q: flow
  - When a force applied to the bearing, the fluid flow resistance changes.
  - A load-balancing pressure differential is developed:

Theory of Hydrostatic Bearings

- The difference in pressure between the upper and lower pads of the bearing is:

  \[
  \Delta P = P_u - P_l = P_s \left( \frac{R_u}{R + R_u} - \frac{R_l}{R + R_l} \right)
  \]

- For a nominal gap \(h\) and small excursions \(\delta\) of the structure:

  \[
  R_u = \frac{\gamma}{(h - \delta)^3} \quad R_l = \frac{\gamma}{(h + \delta)^3}
  \]
Theory of Hydrostatic Bearings

• The difference in pressure across the bearing is:

\[
\Delta P = P_s \gamma \left[ \frac{1}{R(h - \delta)^3 + \gamma} - \frac{1}{R(h + \delta)^3 + \gamma} \right]
\]

Theory of Hydrostatic Bearings

• If the inlet flow resistance \( R \) was zero, the bearing could support no load.
• If the inlet flow resistance was infinite, the bearing could support no load.
• There must be some ideal inlet resistance (compensation) between these two extremes.
• Taking the partial derivative of the pressure difference with respect to the inlet flow resistance;

\[
\frac{\partial \Delta P}{\partial R} = P_s \gamma h^2 \left[ \frac{- (h - 3\delta)}{Rh^2 (h - 3\delta) + \gamma^2} - \frac{(h + 3\delta)}{Rh^2 (h + 3\delta) + \gamma^2} \right]
\]

\[\text{Ignoring all terms with } \delta^2 \text{ and higher terms:}\]
Theory of Hydrostatic Bearings

• The "optimal" inlet flow resistance to maximize load capacity is:
  \[ R = \frac{\gamma}{h^3} \]

➤ A prime issue is the \( h^3 \) term.

○ A capillary restrictor, whose resistance should equal that of the bearing, has

\[ R_{\text{Cpillary}} = \frac{\phi}{D^4_{\text{Cpillary}}} \]

➤ There is the potential for a very high degree of sensitivity to manufacturing tolerances.

Theory of Hydrostatic Bearings

• If the displacement of the bearing is assumed to be a portion of the nominal gap, \( \delta = \alpha h \):

\[ \Delta P = P_s \left( \frac{1}{(1 - \alpha)^3 + 1} - \frac{1}{(1 + \alpha)^3 + 1} \right) \]

• Linearizing about \( \alpha = 0 \):

\[ \Delta P \approx P_u - P_l \approx \frac{P_s}{2 - 3\alpha} - \frac{P_s}{2 + 3\alpha} \approx \frac{3P_s}{2} \alpha \]

• For an opposed pad bearing with supply pressure \( P_s \) and inlet restrictor resistance \( R \), the total flow is just \( Q = P_s/R \).
Theory of Hydrostatic Bearings

• Stiffness is the change in load for a given change in bearing gap \(A \frac{\partial \Delta P}{\partial \delta}\) where \(A_{\text{effective}}\) is the effective bearing area.

\[
K = A_{\text{effective}} \frac{\partial \Delta P}{\partial \delta} = 3P_s A_{\text{effective}} h^3 \left[ \frac{(h - \delta)^2}{[(h - \delta)^3 + h^3]^2} + \frac{(h + \delta)^2}{[(h + \delta)^3 + h^3]^2} \right]
\]

• At maximum load capacity, the bearing stiffness is:

\[
K \approx \frac{3P_s A_{\text{effective}}}{2h} \approx \frac{F_{\text{max}}}{h}
\]

Example

• If \(P = 2\text{MPa} (20 \text{ atm})\), \(a=b=0.05\text{ m}\), \(A_{\text{effective}} = 0.001250\text{ m}^2\) and \(h=10\ \mu\text{ m}\), then

\(K=375\text{ N/ } \mu\text{ m}\) which is a very stiff bearing.
Theory of Hydrostatic Bearings

• The load the bearing can support is \( F = K \delta \), where \( \delta = \alpha h \):

\[
F \approx \frac{3P_s A_{\text{effective}} \alpha}{2} \approx \frac{P_s A_{\text{total}}}{2}
\]

• With \( \alpha = 0.5 \) and a correction factor from Figure 9.2.3 of 0.88, the bearing load capacity is 1650 N (371 lbf).

Bearing Effective Area

• The pocketed region contributes a force equal to:

\[
F_{\text{pocket}} = P_p \left[ (a - 2\ell)(b - 2\ell) + r_p^2(\pi - 4) \right]
\]

• For the land regions not at the corners, the pressure decays linearly and they contribute a force of:

\[
F_{\text{land}} = P_p \ell [a + b - 4(\ell + r_p)]
\]

• For the corner regions, the pressure decays logarithmically and the four corners together act as a single beveled ring:

\[
F_{\text{rc}} = \frac{-2\pi P_p}{\log_e \left( \frac{r_p + \ell}{r_p} \right)} \int_{r_p}^{r_p + \ell} r \log_e \left( \frac{r}{r_p + \ell} \right) \, dr = \pi P_p \left[ \frac{\ell(2r_p + \ell)}{2\log_e \left( \frac{r_p + \ell}{r_p} \right)} - r_p^2 \right]
\]
Bearing Effective Area

• The effective area for the rectangular flat pad pocketed bearing is thus:

\[
A = (a - 2\ell) (b - 2\ell) + \pi r_p^2 (\pi - 4) + \ell [a + b - 4 (\ell + r_p)] + \pi \left[ \frac{\ell (2r_p + \ell)}{2 \log_e \left( \frac{r_p + \ell}{r_p} \right)} - r_p^2 \right]
\]

Theory of Hydrostatic Bearings

• Squeeze film damping
  - The fluid between the lands and the bearing surface is essentially incompressible.
  - When high frequency (velocity) loads are applied, fluid particles must be squeezed out.
  - Viscous shear dominates fluid flow out of the land region.
  - Typical damping coefficients of systems with hydrostatic bearings are $\zeta = 0.1-1$. 

Since most of the damping area in a hydrostatic bearing are rectangular, the damping factor, $b$, is calculated as

$$b = K_s \frac{\mu b w^3 l}{h_o^3}$$

- $\mu$ is the viscosity of the bearing fluid and $K_s$ is a geometric factor related to the bearing.
- Most of the area of a hydrostatic bearing contains small gap regions:
  - Use the entire rectangular region as the squeeze film area.
The constant $K_S$ can be obtained from:

$$K_S = 0.7925 - \frac{1.1005}{e^{w/\ell}} + \frac{0.0216}{w/\ell} + 0.0153 \frac{w}{\ell}$$

Where $b$ is the length of the region, and $l$ is the width.

For $b/l > 10$, $K_S = 1$.

For circular regions, the damping factor $b$ (N/(m/s)) is:

$$b_{	ext{circular}} = \frac{3\pi\mu\left(R_O^2 - R_I^2\right)^2}{2h_s^3}$$

- $\mu$ for IUSO 10 oil at 20°C is 0.010 N-s/m²
- $\mu$ for water at 20°C is 0.001 N-s/m²
Theory of Hydrostatic Bearings

• When the pump is turned off and the bearing allowed to settle:
• The carriage has order of magnitude greater stiffness, and acts like it is a part of the structure.

Types of Compensation

• Opposed pad, capillary restricted bearings are one of the most common hydrostatic bearing designs.
• The flow resistance of a capillary is:

\[ R = \frac{8l\mu}{\pi r_c^4} \]
Types of Compensation

• Typical design:

If the gap should change significantly, however, stiffness can be rapidly lost.

• Effect of mfg. errors on capillary compensated bearing:
Self Compensation

- The bearing gap itself can be used as a means of regulating the flow of fluid to the opposed bearing.

US patent *Self Compensating Hydrostatic Linear Bearing*, #5,104,237, April 14, 1992 (Patents pending in Europe, Asia, and South America).
Self Compensation

- Self compensating bearings' load capacity and stiffness can be theoretically determined:

\[
\Delta P = P_s \left( \frac{1}{(h - \delta)^3} - \frac{1}{(h + \delta)^3} + \frac{1}{\gamma (h - \delta)^3} + \frac{1}{\gamma (h + \delta)^3} \right)
\]

\[
K = 3A_P_s \left( \frac{\gamma}{(h - \delta)^4} - \frac{1}{(h + \delta)^4} \right) + \frac{1}{(h + \delta)^4 \left( \frac{\gamma}{(h - \delta)^3} + \frac{1}{(h + \delta)^3} \right)^2} + \frac{1}{\gamma (h - \delta)^3} + \frac{1}{\gamma (h + \delta)^3} \right)
\]

Self Compensation

- Spreadsheet results for the design of an ideal self compensated bearing:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply Pressure Ps (N/m², psi, atm)</td>
<td>2,028,600</td>
</tr>
<tr>
<td>Viscosity μ (N·s/m²) (water)</td>
<td>0.001</td>
</tr>
<tr>
<td>Nominal bearing gap h (m, in, μm)</td>
<td>0.000010</td>
</tr>
<tr>
<td>Rectangular bearing characteristics (one pad either side of compensator)</td>
<td></td>
</tr>
<tr>
<td>Width a (m, in, mm)</td>
<td>0.0300</td>
</tr>
<tr>
<td>Length b (m, in, mm)</td>
<td>0.0600</td>
</tr>
<tr>
<td>Land width 1 (25% of width) (m, in, mm)</td>
<td>0.0075</td>
</tr>
<tr>
<td>Pocket radius rp (m, in, mm)</td>
<td>0.0040</td>
</tr>
<tr>
<td>Fluid resistance Rbearing (Nsec/m⁵)</td>
<td>6.79E+11</td>
</tr>
<tr>
<td>Effective pad area (cm², in², mm²)</td>
<td>11.14</td>
</tr>
<tr>
<td>Results are for each pad pair:</td>
<td></td>
</tr>
<tr>
<td>Self compensating:</td>
<td>Capillary:</td>
</tr>
<tr>
<td>gamma=Restrictor/Rbearing ()</td>
<td>3</td>
</tr>
<tr>
<td>Load capacity at 50% gap closure (N, lb)</td>
<td>2,006</td>
</tr>
<tr>
<td>Initial stiffness (N/μm, lb/μin)</td>
<td>508</td>
</tr>
<tr>
<td>Stiffness at 25% gap closure (N/μm, lb/μin)</td>
<td>436</td>
</tr>
<tr>
<td>Stiffness at 50% gap closure (N/μm, lb/μin)</td>
<td>184</td>
</tr>
<tr>
<td>Flow (liters per minute)</td>
<td>0.24</td>
</tr>
<tr>
<td>Pump power (Watts)</td>
<td>8.05</td>
</tr>
</tbody>
</table>

16-37
Self Compensation

• One may wish to use self-compensation instead of fixed (capillary or orifice) compensation:

• It makes the system far less sensitive to contamination, especially if water is used:

![Diagram]

- Gooey blob builds at zero velocity point
- Blob eventually gets big and breaks off to clog restrictor

Self Compensation

• It provides greater stiffness and load capacity.

• It makes the system insensitive to manufacturing tolerances:

  ➢ The bearings are self-tuning:

  ☁ The stiffness automatically optimizes itself for the bearing as soon as it is turned on.
  ☁ No manual tuning of capillary or orifice size is required.
Self Compensation

• In addition, it would be preferable to use water (or a water based coolant) instead of oil for the following reasons:
  ➢ Environmentally friendlier.
  ➢ No fire hazard.
  ➢ Greater heat capacity, which minimizes temperature rise.
  ➢ Lower viscosity, which allows for higher speeds.
  ➢ Very tolerant of crosstalk between lubrication and coolant systems.

• In order to use water, the gap must be small to keep flow rates reasonable.

• Self compensated bearings are not significantly affected by large gap variations caused by manufacturing tolerances
  ➢ Thus they are suitable for use with water as a bearing fluid.

Self Compensation

• Effect of mfg. errors on a self-compensated bearing (closer gap, stiffness higher, system stable)
Self Compensation

- Effect of mfg. errors on capillary compensated bearing (closer gap, stiffness lower, system less stable)

Self Compensation

- Conventional hydrostatic bearings require careful hand-tuning to obtain good performance.
- Self compensating bearings will be optimally compensated regardless of the gap at which it is operating.
Self Compensation

• A compensator can be connected to the opposite pocket by external tubes:
  ➢ *Bearings are made as modular pads, vacuumed to a rail, and then potted in-place inside a machine carriage:*

Self Compensation

• A compensator can be connected to the opposite pocket by drilling across the bearing:
  ➢ *Bearings are made as modular blocks which are surrounded by the bearing rails:*

  ![Diagram](image-url)
Aerostatic Bearings

• Typical working spindles on ultra-precision machines are designed with air bearing technology. They are stiff and hold a radial run-out error of less than 25.4 nanometer (1 microinch) as well as handle radial loads of about 300 pounds.

• Hydraulic working spindles were found to have better dampening characteristics than pneumatic working spindles, so their presence became eminent in ultra-precision technology.

Aerostatic Bearing

http://pergatory.mit.edu/perg/research/archive/Kotilainen/hydrobushing.htm
Aerostatic Bearing

HydroBushing™

Aerostatic Bearing Spindle

http://www.nelsonair.com/NA_p rods_spindle.htm

Aerostatic Driving Spindle

Hydrostatic Spindle
Hydrostatic Spindle